PHYSICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 90

Total Marks : 28

Max. Time : 31 min.

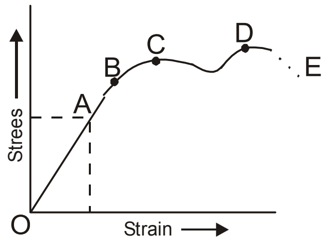
Topic : Variation of Strain with Stress

Type of Questions		M.M., Min.
Single choice Objective ('–1' negative marking) Q.1 to Q.5	(3 marks, 3 min.)	[12,12]
Multiple choice objective ('–1' negative marking) Q.5	(4 marks, 4 min.)	[4, 4]
Subjective Questions ('–1' negative marking) Q.6	(4 marks, 5 min.)	[4, 5]
Match the Following (no negative marking) (2 × 4) Q. 7	(8 marks, 10 min.)	[8, 10]

COMPREHENSION

VARIATION OF STRAIN WITH STRESS

When a wire is stretched by a load, it is seen that for small value of load, the extension produced in the wire is proportional to the load. On removing the load, the wire returns to its original length. The wire regains its original dimensions only when load applied is less or equal to a certain limit. This limit is called elastic limit. Thus, elastic limit is the maximum stress on whose removal, the bodies regain their original dimensions. In shown figure, this type of behavior is represented by OB portion of the graph. Till A the stress is proportional to strain and from A to B if deforming forces are removed then the wire comes to its original length but here stress is not proportional to strain.

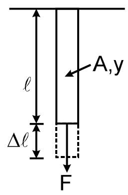


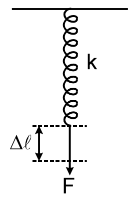
OA → Limit of Proportionality
OB → Elastic limit
C → Yield Point
CD → Plastic behaviour
D → Ultimate point
DE → Fracture

As we go beyond the point B, then even for a very small increase in stress, the strain produced is very large. This type of behaviour is observed around point C and at this stage the wire begins to flow like a viscous fluid. The point C is called yield point. If the stress is further increased, then the wire breaks off at a point D called the breaking point. The stress corresponding to this point is called breaking stress or tensile strength of the material of the wire. A material for which the plastic range CD is relatively high is called ductile material. These materials get permanently deformed before breaking. The materials for which plastic range is relatively small are called brittle materials. These materials break as soon as elastic limit is crossed.

Analogy of Rod as a spring

$$y = \frac{\text{stress}}{\text{strain}} \quad \Rightarrow \quad y = \frac{F\ell}{A\Delta\ell}$$

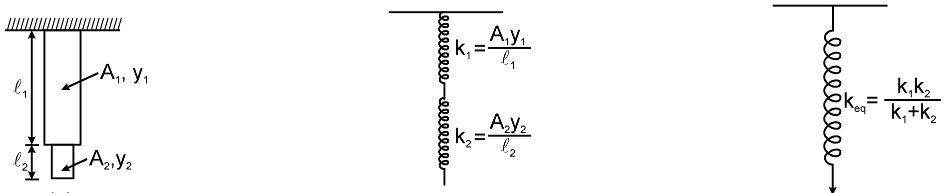




or $\quad F = \frac{Ay}{\ell}\Delta\ell$

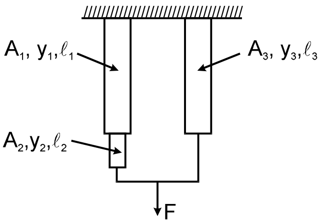
$\frac{Ay}{\ell}$ = constant, depends on type of material and geometry of rod. $F = k\Delta\ell$

where $k = \frac{Ay}{\ell}$ = equivalent spring constant.

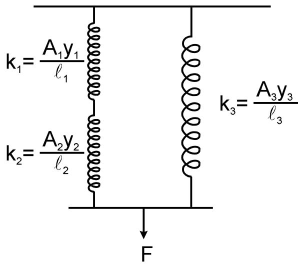


(a) (b) (c)

for the system of rods shown in figure (a), the replaced spring system is shown in figure (b) two spring in series]. Figure (c) represents equivalent spring system.
Figure (d) represents another combination of rods and their replaced spring system.

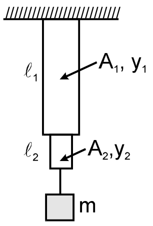


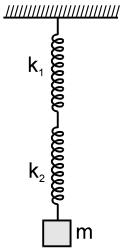
(d)



Illus. 1.

A mass 'm' is attached with rods as shown in figure. This mass is slightly stretched and released whether the motion of mass is S.H.M., if yes then find out the time period.



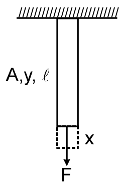


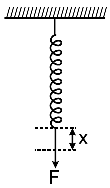
Sol. $k_{eq} = \frac{k_1k_2}{k_1+k_2}$ $\qquad$ $T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$

where $k_1 = \frac{A_1y_1}{\ell_1}$ and $k_2 = \frac{A_2y_2}{\ell_2}$

ELASTIC POTENTIAL ENERGY STORED IN A STRETCHED WIRE OR IN A ROD

Strain energy stored in equivalent spring





$$U = \frac{1}{2}kx^2$$

where $x = \frac{F\ell}{Ay}$, $\quad k = \frac{Ay}{\ell}$ $\qquad U = \frac{1}{2}\frac{Ay}{\ell}\frac{F^2\ell^2}{A^2y^2} = \frac{1}{2}\frac{F^2\ell}{Ay}$.

equation can be re-arranged

$$U = \frac{1}{2}\frac{F^2}{A^2} \times \frac{\ell A}{y}$$ [ℓA = volume of rod, F/A = stress]

$$U = \frac{1}{2}\frac{(\text{stress})^2}{y} \times \text{volume}$$

again, $U = \frac{1}{2}\frac{F}{A} \times \frac{F}{Ay} \times A\,\ell$ $\qquad$ [Strain $= \frac{F}{Ay}$]

$$U = \frac{1}{2}\text{ stress} \times \text{strain} \times \text{volume}$$

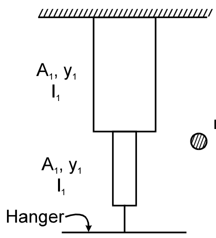
again, $U = \frac{1}{2}\frac{F^2}{A^2y^2}A\ell y$

$$U = \frac{1}{2}y\,(\text{strain})^2 \times \text{volume}$$

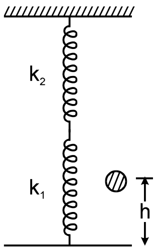
strain energy density $= \frac{\text{strain energy}}{\text{volume}} = \frac{1}{2}\frac{(\text{stress})^2}{y} = \frac{1}{2}y(\text{strain})^2 = \frac{1}{2}\text{ stress} \times \text{strain}$

Illus. 2.

A ball of mass 'm' drops from a height 'h', which sticks to hanger after striking. Neglect over turning, find out the maximum extension in rod. Asumme rod and hanger is massless.



Sol. Applying energy conservation



$$mg\,(h + x) = \frac{1}{2}\,\frac{k_1k_2}{k_1 + k_2}x^2$$

where $k_1 = \frac{A_1y_1}{\ell_1}$ $\quad k_2 = \frac{A_2y_2}{\ell_2}$

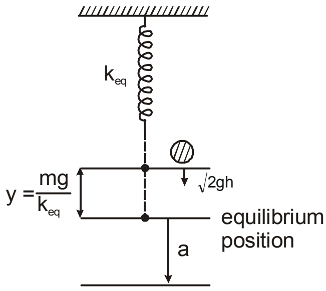
& $\quad K_{eq} = \frac{A_1A_2y_1y_2}{A_1y_1\ell_2 + A_2y_1\ell_1}$

$k_{eq}x^2 - 2mgx - 2mgh = 0$

$$x = \frac{2mg \pm \sqrt{4m^2g^2 + 8mghk_{eq}}}{2k_{eq}} \qquad x_{max} = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2g^2}{k_{eq}^2} + \frac{2mgh}{k_{eq}}}$$

BY S.H.M.

$$w = \sqrt{\frac{k_{eq}}{m}} \qquad v = \omega\sqrt{a^2 - y^2}$$



$$\sqrt{2gh} = \sqrt{\frac{keq}{m}}\sqrt{a^2 - y^2} \Rightarrow \sqrt{\frac{2mgh}{k_{eq}} + \frac{m^2g^2}{k_{eq}^2}} = a$$

max^m extension $= a + y = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2g^2}{k_{eq}} + \frac{2mgh}{k_{eq}}}$

1. If x longitudinal strain is produced in a wire of Young's modulus y, then energy stored in the material of the wire per unit volume is :
(A) yx^2 (B) $2\,yx^2$ (C) $\frac{1}{2}\,y^2x$ (D) $\frac{1}{2}\,yx^2$

2*. A metal wire of length L is suspended vertically from a rigid support. When a bob of mass M is attached to the lower end of wire, the elongation of the wire is ℓ :
(A) The loss in gravitational potential energy of mass M is $Mg\ell$
(B) The elastic potential energy stored in the wire is $Mg\ell$
(C) The elastic potential energy stored in the wire is $\frac{1}{2}Mg\ell$
(D) Heat produced is loss of mechanical energy of system is $\frac{1}{2}Mg\ell$

3*. A metal wire of length L area of cross-section A and Young's modulus Y is stretched by a variable force F such that F is always slightly greater than the elastic force of resistance in the wire. When the elongation of the wire is ℓ :
(A) the work done by F is $\frac{YA^2}{L}$
(B) the work done by F is $\frac{YA\ell^2}{2L}$
(C) the elastic potential energy stored in the wire is $\frac{YA\ell^2}{2L}$
(D) heat is produced during the elongation

4. Two wires of the same material and length but diameter in the ratio 1 : 2 are stretched by the same force. The ratio of potential energy per unit volume for the two wires when stretched will be :
(A) 1 : 1 (B) 2 : 1 (C) 4 : 1 (D) 16 : 1

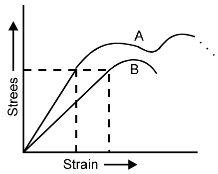
5. The workdone in increasing the length of a one metre long wire of cross-sectional area 1 mm^2 through 1 mm will be ($Y = 2 \times 10^{11}\ Nm^{-2}$) :
(A) 0.1 J (B) 5 J (C) 10 J (D) 250 J

6. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to :

(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{m(YA + kL)}{YAk}}$

(C) $2\pi\sqrt{\frac{mYA}{kL}}$ (D) $2\pi\sqrt{\frac{mL}{YA}}$

7. In the figure shown the strain versus stress graph for two values of young's modulus?



(i) which material is more ductile ? Explain.
(ii) Which material is more brittle? Explain.
(iii) Which material is stronger? Explain.

Answers Key

DPP NO. - 90

1. (D) 2. (A) (C) (D) 3.(B) (C)
4. (D) 5. (A) 6. (B)
7. (i) A(from comprehension)
(ii) B (from comprehension)
(iii) A (A can bear more stress than B before fracture)